Definition and Calculation of the Effective Potential

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Abstract

Two definitions of the effective potential are given, their equivalence is explicitly shown, a sample calculation is then demonstrated for ϕ^4 theory. This calculation is based on the method of steepest descent and is given in the "one-loop approximation".

Introduction

Goldstone's theorem and spontaneous symmetry breakdown are important ingredients in gauge theories and many-body physics.[1,2,3] One method of investigating spontaneous symmetry breakdown is to calculate the function known as the effective potential.[4] For a given field Lagrangian, invariant under some symmetry transformation, the behaviour of the derivative of the effective potential with respect to the "classical field" demonstrates whether the vacuum state of the theory changes under the transformation or remains invariant, i.e. whether the symmetry is spontaneously broken or not.

This paper reviews and brings together the main properties of the effective potential that have been discussed in the literature.[4]. Two definitions of the effective potential are given, the first follows from a variational principle and leads to a physical interpretation, the second is given in terms of a functional formalism, which makes it more suitable for calculations. The equivalence of the two definitions is shown explicitly, and a sample calculation, using the method of steepest descents, is carried out for ϕ^4 theory, in the "one-loop approximation".

II. The Effective Potential

1. The Effective Potential in Quantum Mechanics.

Problems in quantum mechanics can be formulated in terms of a variational principle; a typical problem is: given a Hamiltonian H, to construct a state $|a\rangle$ such that $\langle a|H|a\rangle$ is stationary. There is a constraint:

$$\langle a|a\rangle = 1 \tag{2.1}$$

The standard method of solution is to add a Lagrange multiplier and to vary

$$\langle a|H|a\rangle - E\langle a|a\rangle \tag{2.2}$$

where E is the (unknown) multiplier.

The solution is well-known,

$$H|a\rangle = E|a\rangle \tag{2.3}$$

i.e., $|a\rangle$ is an eigenstate of H with eigenvalue E; the stationary value is

$$\langle a|H|a\rangle = E \tag{2.4}$$

Now suppose that there is a second constraint equation

$$\langle a|\phi|a\rangle = \phi_c \tag{2.5}$$

where ϕ is some operator, for example such a constraint could be used in applying the variational method to find the ground state of the Heisenberg ferromagnet, (2.5) would hen be interpreted to mean that the problem is con-strained by the requirement of a non-vanishing ground-state expectation value for the spontaneous magnetisation.

In this case the quantity to be varied is

$$\langle a|H|a\rangle - \varepsilon\langle a|a\rangle - J\langle a|\phi|a\rangle \tag{2.6}$$

here ε and J are Lagrange multipliers.

The solution is

$$(H - J\phi)|a\rangle = \varepsilon|a\rangle \tag{2.7}$$

.e., $|a\rangle$ is an eigenstate of the modified Hamiltonian $- J\phi$ with eigenvalue ε; the stationary value is

$$\langle a|H|a\rangle = \varepsilon + J\phi_c \tag{2.8}$$

From (2.7), ε depends on J and (2.8) shows that $a|H|a\rangle$ is a function of J and ϕ_c:

$$\langle a|H|a\rangle = F(J, \phi_c) \tag{2.9}$$

The stationary condition on $\langle a|H|a\rangle$ implies

$$\delta F(J,\phi_c) = 0 = \frac{\partial\varepsilon}{\partial J}\delta J + \phi_c\,\delta J + J\,\delta\phi_c \tag{2.10}$$

owever, $\delta\phi_c = 0$ and eq. (2.8) give

$$\phi_c = -\frac{\partial\varepsilon}{\partial J} \tag{2.11}$$

Eq. (2.11) may be used to express J as a function of $_c$ and then eq. (2.8) may be written entirely as a function f ϕ_c:

$$\langle a|H|a\rangle = V(\phi_c) \tag{2.12}$$

The function on the right hand side of eq. (2.12) is called the <u>effective potential</u>.

In words, the effective potential is the expectation value of the Hamiltonian in a state $|a\rangle$ for which $\langle a|H|a\rangle$ is stationary and which is constrained to satisfy $\langle a|a\rangle = 1$ and $\langle a|\phi|a\rangle = \phi_c$ where ϕ is some (relevant) operator.

A similar definition can be given for a quantum field theory, where ϕ is now a field and the Hamiltonian becomes a Hamiltonian denstiy.

This definition may be compared to a classical field theory where the ordinary potential is the energy density for a state in which the classical field assumes the value ϕ.

An important result which follows from (2.12) is that

$$\frac{\partial V}{\partial\phi_c} = 0 \tag{2.13}$$

for $\langle a|H|a\rangle$ to be stationary.

The above definition of the effective potential for quantum field theory does not include a prescription for its evaluation. A method of calculation can be established, this requires a review of some standard functional techniques. The point of this review is to define a quantity which may calculated and which is equivalent to the effective tential as defined in eq. (2.8) and eq. (2.12).

The Generating Functional.

Consider the theory of a single scalar field with sociated Lagrangian density $\mathcal{L}(\phi, \partial_\mu \phi)$ given by

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi)^2 - \tfrac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4 - J\phi \tag{2.14}$$

The Lagrangian density has been modified to include the rm $J(x)\phi(x)$ where $J(x)$ is some external source, a c-number nction of space and time.

The connected generating functional $W[J]$ is defined [4]

$$e^{iW[J]} \equiv \langle 0^+ | 0^- \rangle_J \tag{2.15}$$

e., it is the vacuum-to-vacuum amplitude in the presence the source $J(x)$; the square brackets indicate that W is functional of J.

W may be expanded in a functional Taylor series:

$$W = \sum_n \frac{1}{n!}\int d^4x_1 \cdots d^4x_n \, G^{(n)}(x_1 \ldots x_n) J(x_1) \cdots J(x_n) \tag{2.16}$$

iere $G^{(n)}$ can be identified as the sum of all connected ynman diagrams with n external lines.

The effective action $\Gamma[\phi_c]$ is defined by a functional egendre transformation:

$$\Gamma[\phi_c] \equiv W[J] - \int d^4x \, J(x)\phi_c(x) \tag{2.17}$$

where $\phi_c(x)$ -- the "classical field" -- is given by

$$\phi_c(x) \equiv \frac{\delta W[J]}{\delta J(x)} \tag{2.18}$$

Eq. (2.18) is used to determine J as a functional of ϕ_c and then J in eq. (2.17) is replaced to give Γ as a functional of ϕ_c.

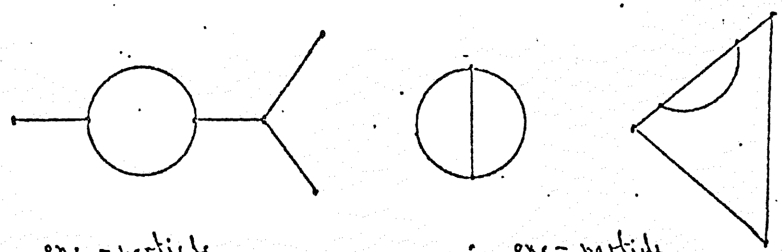
From eq. (2.17),

$$\frac{\delta\Gamma}{\delta\phi_c(x)} = -J(x) \tag{2.19}$$

The functional series for Γ in terms of $\phi_c(x)$ is

$$\Gamma[\phi_c] = \sum_n \frac{1}{n!}\int d^4x_1 \cdots d^4x_n \, \Gamma^{(n)}(x_1 \ldots x_n)\phi_c(x_1) \cdots \phi_c(x_n) \tag{2.20}$$

where the $\Gamma^{(n)}$ are the 1PI Green's functions; $\Gamma^{(n)}$ is the sum of all 1PI Feynman diagrams with n internal lines. A 1PI (one-particle-irreducible) diagram is a connected diagram that cannot be disconnected by cutting a single internal line, such a diagram is evaluated with no propagation on the external lines (see figure 1).



one-particle reducible graph

one-particle irreducible graphs

fig. 1.

$\Gamma[\phi_c]$ may also be expanded in powers of the derivative ϕ_c around the point ϕ_c = constant:

$$\Gamma[\phi_c] = \int d^4x \left[-V(\phi_c) + \tfrac{1}{2}(\partial_\mu \phi_c)^2 Z(\phi_c) + \cdots \right] \tag{2.21}$$

ere the function $V(\phi_c)$, as the notation suggests, will rn out to be the effective potential defined in eq. (2.8) d eq. (2.12).

Comparison of the two series for Γ leads to:

$$V(\phi_c) = -\sum_{n=0}^{\infty} \frac{1}{n!} (\phi_c)^n \, \Gamma^{(n)}(0, \cdots, 0) \tag{2.22}$$

e., the n^{th} derivative of $V(\phi_c)$ is the sum of all 1PI aphs with n vanishing momenta.

The next step is to show that the function $V(\phi_c)$ fined in eq. (2.21) is equivalent to the effective pot- tial defined in eq. (2:8) and eq. (2.12). This can be hieved by noting that W[J] may be expanded in powers of e derivatives of J around the point J = constant:

$$W[J] = \int d^4x \left[-\varepsilon(J) + \tfrac{1}{2}(\partial_\mu J)^2 \chi(J) + \cdots \right] \tag{2.23}$$

If J and ϕ_c are constant, then equations (2.17), (2.21) d (2.23) imply

$$V(\phi_c) = \varepsilon(J) + J\phi_c \tag{2.24}$$

This equation is identical to eq. (2.8) provided that ne terms $\varepsilon(J)$ and ϕ_c defined in eq. (2.18) and eq. (2.23)

can be shown to be the same as the terms ε and ϕ_c defined in eq. (2.7) and eq. (2.11); if this is the case, then the function $V(\phi_c)$ defined in eq. (2.21) is equal to $\langle a|H|a\rangle$, i.e., it is identical to the effective potential.

To prove all this, it is first necessary to give a physical interpretation to the ε of eq. (2.23). This may be done as follows:

A simple system which needs only one co-ordinate q to describe it has an associated state vector $|q,t\rangle$, where t is the time. If an external source J is added, as before, to the Lagrangian for the system then it can be shown [5] that the ground-state to ground-state amplitude for the system is given by

$$\langle 0^+|0^-\rangle_J \propto \lim_{\substack{t \to +i\infty \\ t' \to -i\infty}} \langle q',t'|q,t\rangle_J \quad , \; t' > t \tag{2.25}$$

If $|n\rangle$ are the energy eigenstates of the system, then

$$|q\rangle = \sum_n a_n |n\rangle \quad , \quad |q'\rangle = \sum_n b_n |n\rangle \tag{2.26}$$

where

$$|q,t\rangle = e^{iHt}|q\rangle \tag{2.27}$$

The amplitude $\langle q',t'|q,t\rangle$ becomes

$$\begin{aligned} \langle q',t'|q,t\rangle &= \langle q'| e^{iH(t-t')}|q\rangle \\ &= \sum_n a_n b_n^* e^{iE_n(t-t')} \\ &= a_0 b_0^* e^{iE_0(t-t')} , \quad \text{as } t \to i\infty, \; t' \to -i\infty \end{aligned} \tag{2.28}$$

ince E_0 is the lowest eigenvalue.

The functional W[J] is defined in terms of the ground-tate to ground-state amplitude through eq. (2.15); comb-nation of eq. (2.15) and the result, eq. (2.28) leads to

$$W[J] \sim E_0(t-t') \qquad (2.29)$$

For a constant J, eq. (2.23) gives

$$W[J] = -\int d^4x\, \mathcal{E}(J) \qquad (2.30)$$

d eq. (2.30) implies

$$\int d^3x\, \mathcal{E}(J) = E_0 \quad , \qquad \text{with } t'-t = \int dx^0 \qquad (2.31)$$

e.,

J) may be identified as an energy density and is thus uivalent to the ε used in the definition of the effective tential for a field theory. Again, because of eq. (2.30) is immediate that the definition of ϕ_c in eq. (2.18) is entical to that of eq. (2.11).

Finally, then, the function $V(\phi_c)$ occuring in the pansion (2.21) of the effective action is in fact the fective potential defined by equations (2.8) and (2.12); e function ϕ_c defined formally in eq. (2.18) is, from . (2.11) and eq. (2.5), seen to be nothing but the pectation value of the field ϕ in the state $|a\rangle$, where is usually the vacuum state.[6]

The point behind all this formalism is that it is readily applicable to spontaneous symmetry breakdown. As a specific example, the Lagrangian density (2.14) is invariant under the transformation $\phi \to -\phi$ provided that J is set equal to zero; spontaneous symmetry breaking occurs if ϕ develops a non-zero vacuum expectation value, i.e., ϕ_c as defined in eq. (2.18) is non-zero; since the source J is zero this situation corresponds to, from eq. (2.18),

$$\frac{\delta\Gamma}{\delta\phi_c(x)} = 0 \quad , \quad \phi_c \neq 0 \qquad (2.32)$$

It is usual to take the vacuum expectation value to be translation-invariant and equations (2.21) and (2.32) immediately lead to

$$\frac{dV}{d\phi_c} = 0 \quad , \quad \phi_c \neq 0 \qquad (2.33)$$

The value of ϕ_c for which V is stationary is the expectation value of ϕ in the new vacuum; the stationary point in fact must be minimum to ensure stability under small external forces.

Thus, spontaneous symmetry breakdown can be investigated by examining the minima of $V(\phi_c)$.

Note that the result (2.33) is a restatement of the variational condition (2.13) and the constraint (2.5).

. The Loop Expansion.

What has been established so far is a definition of the ffective potential in terms of the functionals $\Gamma[\phi_c]$ and [J] which generate Feynman diagrams. It is obvious from q. (2.22) that the evaluation of $V(\phi_c)$ involves an infinite ummation of graphs; the easiest way to proceed is to estblish an approximation method for $V(\phi_c)$. One such method s to write the vacuum-to-vacuum amplitude as a Feynman ath-integral. [7]

$$0^+|0^-\rangle_J = e^{iW[J]} = \frac{\int[d\phi]\exp\{i\int d^4x[\mathcal{L}(\phi,\partial_\mu\phi)+J\phi]\}}{\int[d\phi]\exp\{i\int d^4x\,\mathcal{L}(\phi,\partial_\mu\phi)\}} \qquad (2.34)$$

here, for the scalar field example,

$$\mathcal{L}(\phi,\partial_\mu\phi) = \tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4 \qquad (2.35)$$

e notation $[d\phi]$ indicates that the integration is unctional.

$V(\phi_c)$ is defined in terms of $\Gamma[\phi_c]$ which itself is etermined from W[J], and W[J] is given by the path-integral; suitable expansion of the path-integral which then gives series for $V(\phi_c)$. Such an expansion is achieved by ntroducing a parameter α into the Lagrangian density $(\phi,\partial_\mu\phi)$ through the definition

$$\mathcal{L}(\phi,\partial_\mu\phi,\alpha) = \frac{1}{\alpha}\mathcal{L}(\phi,\partial_\mu\phi) \qquad (2.36)$$

d then expressing the exponent of the path integral as a

power series in α. Since W[J] generates Feynman diagrams and is related to the path-integral through eq. (2.34), the power series in α should correspond to some summation of these graphs. In fact, the α-expansion is equivalent to a loop expansion, [8] i.e., first summing all the tree graphs -- the Feynman diagrams with no closed loops -- then summing the diagrams with one closed loop, those with two closed loops, etc. This is easy to show. In any diagram, the propagator carries a factor of α because it is the inverse of the differential operator occuring in $\mathcal{L}$, also, a vertex has a factor of α^{-1}. For a graph with I internal lines and V vertices the power P of α for the graph is

$$P = I - V \qquad (2.37)$$

In a diagram, the independent integration momenta give the number of loops, this is because there is an integration momentum for every internal line, the δ-function from each vertex reduces the independent momenta by one, (except for one δ-function due to energy-momentum conservation.). If L is the number of loops, then

$$L = I - V + 1 \qquad (2.38)$$

and so

$$P = L - 1 \qquad (2.39)$$

which shows that the power series expansion in α is equivent to the loop expansion.

It is usual to set the parameter $\alpha = \hbar$, but the mallness" of $\hbar$ has nothing to do with the validity of the wer series, indeed α can be set equal to unity after the pansion. What is important is the fact that the total grangian density is multiplied by α; so that the calcu- tion of the effective potential is not affected by any ift of the fields.

The Method of Steepest Descent.[7]

When $\alpha = \hbar$ is introduced into eq. (2.34) it becomes

$$e^{iW[J]/\hbar} = \frac{\int[d\phi]\exp\left\{\frac{i}{\hbar}\int d^4x\left[\mathcal{L}(\phi,\partial_\mu\phi)+J\phi\right]\right\}}{\int[d\phi]\exp\left\{\frac{i}{\hbar}\int d^4x\,\mathcal{L}(\phi,\partial_\mu\phi)\right\}} \tag{2.40}$$

The method of steepest descent is a standard way of rforming the $\hbar$-expansion: The exponent in the numerator the right-hand side of eq. (2.40) is expanded around the int $\phi_0[J]$ at which it is stationary. (The denominator included to give $W[0] = 0$).

The calculation is straightforward for the field theory scribed by eq. (2.14). The stationary value ϕ_0 satisfies e equation of motion:

$$-(\Box+\mu^2)\phi_0 - \frac{1}{3!}\lambda\phi_0^3 = -J \tag{2.41}$$

$J(x)$ may be taken to be zero at infinity, ϕ_0 is then ique if it also is zero at infinity,

$$\phi_0[J] = 0 \quad \text{for} \quad J = 0 \tag{2.42}$$

The action $I[\phi]$ is defined by

$$I[\phi] \equiv \int d^4x\left[\tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}\mu^2\phi^2 - \tfrac{\lambda}{4!}\phi^4 + J\phi\right] \tag{2.43}$$

This is expanded around the point ϕ_0 by setting $\phi = \phi_0 + \tilde{\phi}$,

$$I[\phi] = I[\phi_0] + \int d^4x\left[\tfrac{1}{2}(\partial_\mu\tilde{\phi})^2 - \tfrac{1}{2}(\mu^2+\tfrac{\lambda}{2}\phi_0)\tilde{\phi}^2 + \tfrac{\lambda}{3!}\phi_0\tilde{\phi}^3 + \tfrac{\lambda}{4!}\tilde{\phi}^4\right] \tag{2.44}$$

The linear terms have been eliminated by using eq. (2.41).

The $\hbar$-expansion for $W[J]$ is then

$$W[J] = W_0[J] + \hbar W_1[\phi_0] + \hbar^2 W_2[\phi_0] + \cdots \tag{2.45}$$

where

$$W_0[J] = -I(\phi_0) \tag{2.46}$$

and

$$\exp\left\{\frac{i}{\hbar}(W[J]-W_0[J])\right\} = \frac{\int[d\phi]\exp\left\{\frac{i}{\hbar}\int d^4x\left[\tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}(\mu^2+\tfrac{1}{2}\lambda\phi_0^2)\phi^2 + \tfrac{\lambda}{3!}\phi_0\phi^3 + \tfrac{\lambda}{4!}\phi^4\right]\right\}}{\int[d\phi]\exp\left\{\frac{i}{\hbar}\int d^4x\left[\tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}\mu^2\phi^2 + \tfrac{\lambda}{4!}\phi^4\right]\right\}} \tag{2.47}$$

If ϕ is rescaled to $\hbar^{1/2}\phi$, eq. (2.47) becomes

$$\exp\left\{\frac{i}{\hbar}(W[J]-W_0[J])\right\} = \frac{\int[d\phi]\exp\left\{i\int d^4x\left[\tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}(\mu^2+\tfrac{1}{2}\lambda\phi_0^2)\phi^2 + \hbar^{\frac{1}{2}}\tfrac{\lambda}{3!}\phi_0\phi^3 + \hbar\tfrac{\lambda}{4!}\phi^4\right]\right\}}{\int[d\phi]\exp\left\{i\int d^4x\left[\tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}\mu^2\phi^2 + \hbar\tfrac{\lambda}{4!}\phi^4\right]\right\}} \tag{2.48}$$

The quadratic terms in the exponents of the numerator and denominator determine $W_1[\phi_0]$, the functional integration of these terms may be carried out by writing them in the orm $\phi M \phi$ and using the result [9]

$$\int [d\phi] \exp\{i(\phi M \phi)\} = (\det M)^{-\frac{1}{2}} \tag{2.49}$$

This gives

$$W_1 = -\tfrac{1}{2} \log \det \left\{ \frac{K_{xy}(\phi_0)}{K_{xy}(0)} \right\} \tag{2.50}$$

$$= -\tfrac{1}{2} \operatorname{Tr} \log \left\{ \frac{K_{xy}(\phi_0)}{K_{xy}(0)} \right\} \tag{2.51}$$

ıere

$$K_{xy}(\phi_0) = (\partial_x \partial_y + \mu^2 + \tfrac{1}{2}\lambda \phi_0^2)\, \delta^4(x-y) \tag{2.52}$$

The higher-order terms W_2, W_3, etc. can be read .rectly from eq. (2.48).

The series for $\Gamma[\phi_c]$ is

$$\Gamma[\phi_c] = \Gamma_0[\phi_c] + \hbar \Gamma_1[\phi_c] + \hbar^2 \Gamma_2[\phi_c] + \cdots \tag{2.53}$$

From equations (2.18), (2.41), (2.45), and (2.46),

$$\phi_c \equiv \frac{\delta W[J]}{\delta J} = \frac{\delta W_0[J]}{\delta J} + O(\hbar) = \phi_0 + O(\hbar) \tag{2.54}$$

that

$$\Gamma_0[\phi_c] = -\int [\tfrac{1}{2}(\partial_\mu \phi_c)^2 - \tfrac{1}{2}\mu^2 \phi_c^2 + \tfrac{\lambda}{4!}\phi_c^4]\, d^4x$$
$$= -I[\phi_c] - \int J \phi_c\, d^4x \tag{2.55}$$

Eq. (2.54) shows that ϕ_c may be written as

$$\phi_c = \phi_0 + \hbar \tilde{\phi}_c \tag{2.56}$$

and then

$$W[J] - \int d^4x\, J\phi_c - \Gamma_0[\phi_c]$$
$$= -\int d^4x\, J\phi_c - \Gamma_0[\phi_c] + \hbar W_1[\phi_0] + \hbar^2 W_2[\phi_0] + \cdots$$
$$= \hbar W_1[\phi_0] + \hbar^2 W_2[\phi_0] + \cdots - I[\phi_0] + I[\phi_c]$$
$$= \hbar W_1[\phi_c - \hbar\tilde{\phi}_c] + \hbar^2 W_2[\phi_c] + O(\hbar^3) \tag{2.59}$$

It follows from eq. (2.53) and eq. (2.57) that

$$\Gamma_1[\phi_c] = W_1[\phi_c] = -\tfrac{1}{2} \operatorname{Tr} \left\{ \frac{K_{xy}(\phi_c)}{K_{xy}(0)} \right\} \tag{2.58}$$

The h-series for $V(\phi_c)$ is

$$V(\phi_c) = V_0(\phi_c) + \hbar V_1(\phi_c) + \hbar^2 V_2(\phi_c) + \cdots \tag{2.59}$$

where each term can be found from its corresponding term in the series (2.53) for $\Gamma[\phi_c]$ by first setting ϕ_c = constant and separating off the factor $\int d^4x$.

From eq. (2.55),

$$V_0(\phi_c) = \tfrac{1}{2}\mu^2 \phi_c^2 + \tfrac{\lambda}{4!}\phi_c^4 \tag{2.60}$$

-- the tree graph contribution is simply the negative sum of the non-derivative terms in the Lagrangian density .

$V_1(\phi_c)$ is obtained from $\Gamma_1[\phi_c]$; for a constant ϕ_c:

$$K_{xy}(\phi_c) = (\partial_x \partial_y + \mu^2 + \tfrac{1}{2}\lambda \phi_c^2)\, \delta^4(x-y)$$
$$= \int \frac{d^4k}{(2\pi)^4} (k^2 - \mu^2 - \tfrac{1}{2}\phi_c^2) \exp[ik(x-y)] \tag{2.61}$$

and then

$$\frac{1}{2}\,\mathrm{Tr}\log\frac{K(\phi_c)}{K(0)} = \frac{1}{2}\int d^4x\int\frac{d^4k}{(2\pi)^4}\log\left(\frac{k^2-\mu^2-\frac{1}{2}\lambda\phi_c^2}{k^2-\mu^2}\right) \qquad (2.62)$$

his gives

$$V_1(\phi_c) = \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\log\left[1+\frac{\frac{\lambda}{2}\phi_c^2}{k^2-\mu^2}\right] + A\phi_c^2 + B\phi_c^4 \qquad (2.63)$$

here the counterterms $A\phi_c^{\ 2}$ and $B\phi_c^{\ 4}$ are included since the ntegral is ultraviolet divergent.

In terms of $V(\phi_c)$, the renormalisation conditions are[10]

$$\left.\frac{d^2V(\phi_c)}{d\phi_c^2}\right|_{\phi_c=0} = \mu^2 \quad , \quad \left.\frac{d^4V(\phi_c)}{d\phi_c^4}\right|_{\phi_c=0} = \lambda \qquad (2.64)$$

These are already satisfied by $V_0(\phi_c)$; A and B are etermined by applying eq. (2.64) to eq. (2.63).

Finally,

$$V_1(\phi_c) = \frac{\mu^4}{4(4\pi)^2}\left[(x+1)^2\log(x+1) - \left(\tfrac{3}{2}x^2+x\right)\right] \qquad (2.65)$$

here

$$x = \frac{\lambda}{2\mu^2}\phi_c^2 \qquad (2.66)$$

uation (2.65) shows that the vanishing of the derivative with respect ϕ_c corresponds to a zero value for ϕ_c, i.e. there is no spontaneous mmetry breakdown to this order.

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